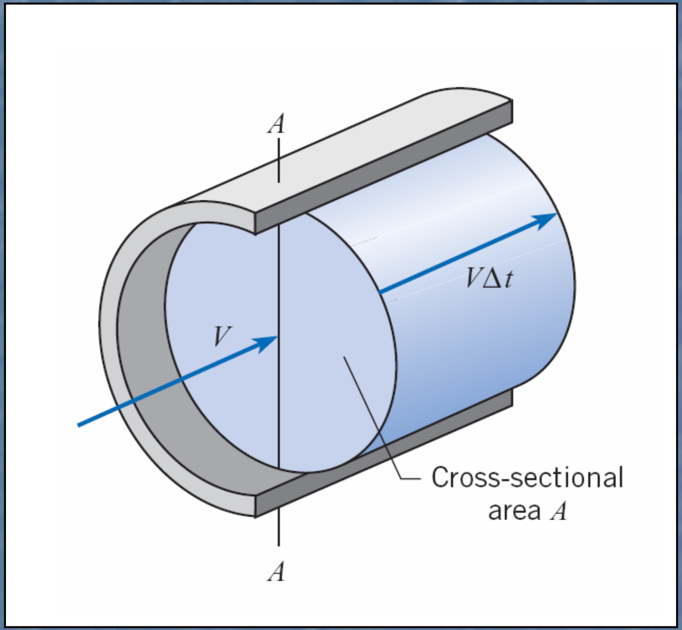
CONTROL VOLUME APPROACH

MASS FLOW RATE

Volume Flow Rate $\dot{Q} = AV$

Mass Flow Rate $\dot{m} = \rho\dot{Q} = \rho AV$

Note: Velocity is constant across the section (A-A)

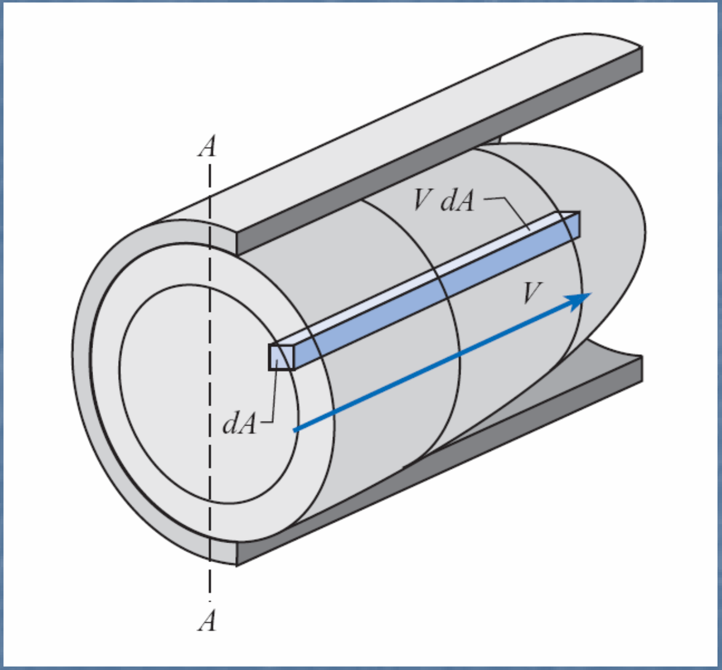


CONTROL VOLUME APPROACH

a. Considering the velocity across the section A-A is variable
b. Considering the density is constant across the section A-A
c. Velocity is always normal to cross sectional area

Volume Flow Rate $\dot{Q} = \int_A V dA$

Mass Flow Rate $\dot{m} = \int_A \rho V dA = \rho \int_A V dA = \rho \dot{Q}$



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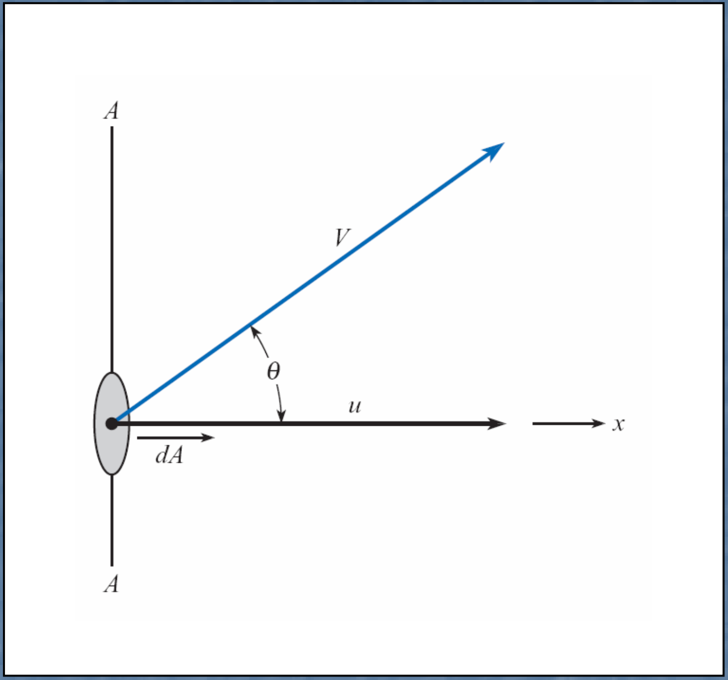


In the above Figure, the flow rate is given as:

$$\dot{m} = \int_A \rho V \cos\theta \, dA$$

Where the differential area dA is normal to velocity flow

MEAN VELOCITY $$\bar{V} = \frac{\dot{Q}}{A}$$

Example (5.1)

Air that has a mass density of 1.24 kg/m^3 (0.00241slugs/ft^3) flows in a pipe with a diameter of 30 cm (0.984 ft) at a mass rate of flow of 3 kg/s (0.206 slugs/s). What are the mean velocity of flow in this pipe and the volume rate of flow?

Solution

$$\dot{m} = \rho Q \qquad \text{or} \qquad Q = \frac{\dot{m}}{\rho} = 2.42 \text{ m}^3/\text{s} \;\; (85.5 \text{ cfs})$$

$$V = \frac{Q}{A} = \frac{2.42}{(\frac{1}{4}\pi) \times (0.30)^2} = 34.2 \text{ m/s} \; (112 \text{ ft/s}) \qquad \triangleleft$$

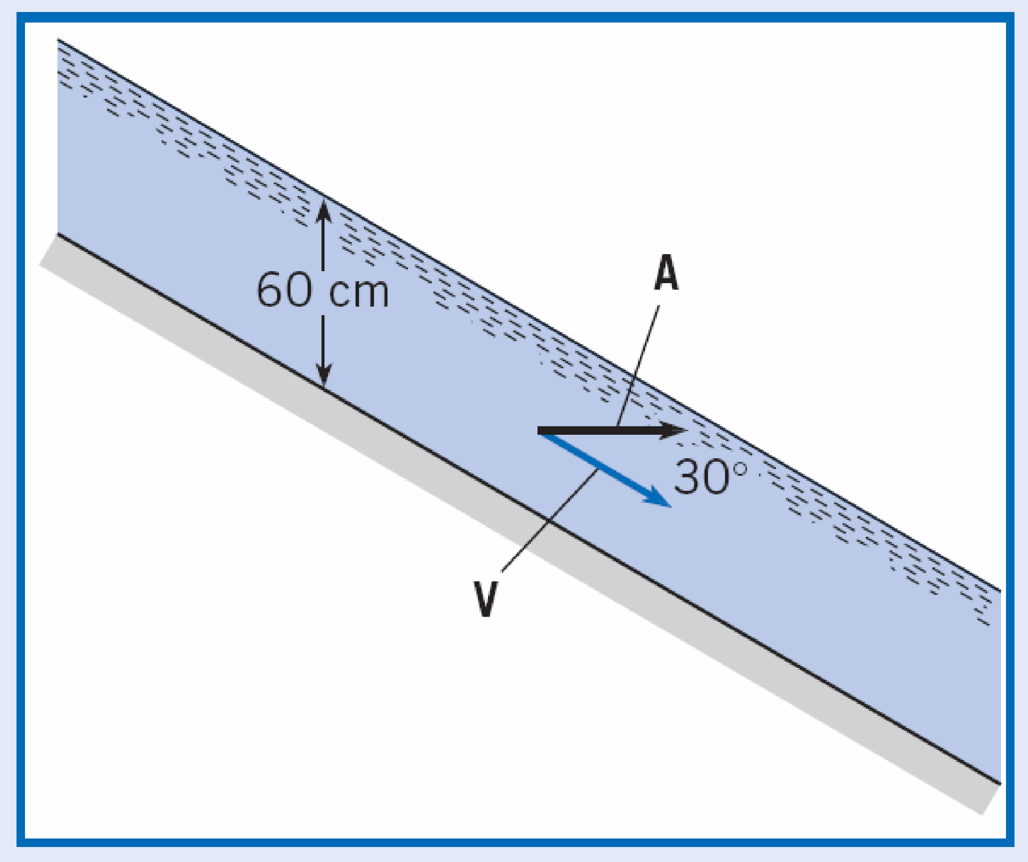
Example (5.2)

Water flows in a channel that has a slope of 30°. If the velocity is assumed to be constant, 12 m/s, and if a depth of 60 cm measured along a vertical line, what is the discharge per meter of width of the channel?

Given: V=12m/s

d=60cm

Find: rate of flow per meter of width



Example (5.2)

Solution The discharge in 1 meter of width is

$$Q = V\cos 30° \times A$$
$$= 12\text{ m/s} \times \cos 30° \times (0.6\text{ m} \times 1\text{ m})$$
$$= 62.4\text{ m}^3/\text{s}$$

The discharge per unit width is usually designated as q and is obtained by dividing through by the width, w, which in this case is 1 meter, so

$$q = \frac{Q}{w} = 62.4\text{ m}^2/\text{s}$$

CONTROL VOLUME APPROACH

END OF LECTURE (1)